Name:_____
SL Physics
Measurements and Errors

Since measurements are never perfect, experimental data must consider the possibility of an error occurring while measuring the data. Errors are dependent on the type of device being used; either digital or analog.

Digital Devices:

- Electronically determines and electronically displays data measurements
- Examples:
  - Electronic Gram Scale
  - Multimeter(Voltmeter + Ammeter + Ohmeter)
  - Stopwatch
- Error for a digital device: $\pm$ the smallest increment the device can measure.

Example 1:

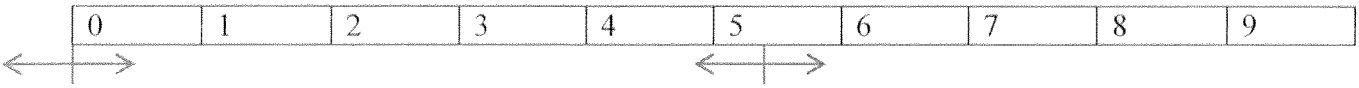
A stopwatch measures time to the nearest hundredth of a second. During an experiment, a student measures a time of 32.14 seconds. The student records this data with the error as: $32.14 \pm 0.01\ s \quad (equivalent\ to \quad a \pm \Delta a\ \ where\ a\ is\ 32.14\ s\ and\ \Delta a\ is\ 0.01s)$

- The plus-or-minus sign ($\pm$) denotes the error which is referred to as Δa
- Since the stopwatch measures to the nearest hundredth of a second, the smallest increment is one one-hundredth (1/100) of a second. The error is therefore one one-hundredth as well (0.01)

Analog Devices:

- Whoever is conducting the experiment must determine the measurement. Generally does not require electricity to determine or display the measurement
- Examples:
  - Ruler
  - Measuring Cup/Beaker/etc.
  - Clock (With a second hand)
- Error for an analog device: half the smallest increment
- When measurements are made with an analog device, usually two measurements must be made. The first measurement is the starting position; the second is the ending position. Both measurements must be recorded with the proper analog error.
- Measurements are made to one tenth the smallest increment, so the last digit is "guessed".

Ruler (cm):



First Measurement (0.0 cm) Second "guessed" measurement (5.4 cm)

Example 2:

A student uses a ruler to measure the length of an object during an experiment. The ruler's smallest increment is 1 cm. The student measures the starting position to be at 0.0 cm. The end position is at 5.4 cm. The student records the two pieces of data as (See ruler above):

$$0.0 \pm 0.5\ cm$$
$$5.4 \pm 0.5\ cm$$

- Even though the ruler only measures as small as centimeters, the student "guesses" (one more decimal point) to the nearest 0.1 cm, using his or her best judgment
- Also, the student determines the error to be half the smallest increment, which would be $\pm 0.5\ cm$

The measurements taken from an experiment and recorded directly are called *raw data* or *unprocessed data*. This data has not been used in calculations yet. While raw data is necessary in receiving results from an experiment, the data often times requires processing for the analysis. *Processed data* is the result of calculations being made with raw data, in order to better understand the significance of the experiment. Since the raw data has an error associated with it, the processed data must also have an error, but also since calculations are being made to the data, the errors must also be operated on and processed with the data.

*Constants and accepted values are considered "perfect" and thus have no error or an error of ± 0.0

<u>Addition and Subtraction with Error:</u>

When adding or subtracting two numbers, add them as one would normally, considering significant digits and **always add** the errors together. Addition and subtraction with error follows the following relationship:

Measurements:

$$a \pm \Delta a$$
$$b \pm \Delta b$$

Sum:

$$c = a + b$$
$$\Delta c = \Delta a + \Delta b$$

Difference:

$$c = a - b$$
$$\Delta c = \Delta a + \Delta b$$

*Whether the operation is addition or subtraction, the errors always add together

Example 3:

A student measures two different lengths and needs to find the sum of the two. The measurements are:

$$a = 16.4 \pm 0.2\ cm$$
$$b = 25.1 \pm 0.1\ cm$$

Let's say we want to find c, the sum of these two numbers. The processed sum of the two pieces of comes out to:

$$c = a + b = 16.4\ cm + 25.1\ cm = 45.1\ cm$$

The error of this processed data is:

$$\Delta c = \Delta a + \Delta b = 0.2\ cm + 0.1\ cm = 0.3\ cm$$

The data should be finally recorded as:

$$45.1 \pm 0.3\ cm$$

Example 4:

In example 2 a student began measuring the length of an object by recording a starting and ending position on a ruler. In order to determine the actual length of the object, the two positions need to be subtracted. The two data values were:

$$0.0 \pm 0.5\ cm$$
$$48.2 \pm 0.5\ cm$$

The processed difference comes out to:

$$48.2\ cm - 0.0\ cm = 48.2\ cm$$

The error of this processed data is:

$$0.5\ cm + 0.5\ cm = 1.0\ cm$$

The final data should be recorded as:

$$48.2 \pm 1.0\ cm$$

*Make sure that the error has the **SAME number of decimal places** as the data. The error for this previous example must be $\pm 1.0\ cm$ <u>not</u> $\pm 1\ cm$

Multiplication and Division with Error:

Multiplying and Dividing data with errors is slightly more involved than adding and subtracting. Instead of the errors adding, the percent errors add. Multiplication and Division with error follows the following relationship:

Measurements:

$$a \pm \Delta a$$
$$b \pm \Delta b$$

Multiplication:

$$c = a * b$$
$$\frac{\Delta c}{c} = \frac{\Delta a}{a} + \frac{\Delta b}{b}$$

Division:

$$c = \frac{a}{b}$$
$$\frac{\Delta c}{c} = \frac{\Delta a}{a} + \frac{\Delta b}{b}$$

*For multiplication and division, make sure the error is calculated using the above ratios, not by simply adding the errors

Example 5:

A student determines the acceleration of a cart rolling down a ramp to be $2.65 \pm 0.12 \frac{m}{s^2}$. The student also determines the mass of the cart to be $.1513 \pm 0.0001\ kg$. The student must use this information to calculate the force acting on the cart. The data values are:

$$2.65 \pm 0.12 \frac{m}{s^2}$$
$$.1513 \pm 0.0001\ kg$$

The product of the two data values:

$$F = ma$$
$$2.65 \frac{m}{s^2} \times .1513\ kg = 0.401\ N$$

Error of the processed data:

$$\frac{\Delta F}{F} = \frac{\Delta a}{a} + \frac{\Delta m}{m}$$

$$\frac{\Delta a}{a} = \frac{0.12\ \frac{m}{s^2}}{2.65\ \frac{m}{s^2}} = 0.04528$$

$$\frac{\Delta m}{m} = \frac{0.0001\ kg}{.1513\ kg} = 0.0006609$$

$$\frac{\Delta F}{0.401\ N} = 0.04528 + 0.0006609$$

$$\Delta F = 0.018\ N$$

Final data should be recorded as:

$$0.401 \pm 0.018\ N$$

*Remember that the error needs to be rounded so that the error has the same number of decimal places as the data value

Example 6:

A student wants to determine the electric field between two parallel plates. Through an experiment the student determines the force on an electrically charged oil drop between the two plates is $4.50 \times 10^{-3} \pm 0.10 \times 10^{-3}\ N$. The student also determines that the charge of oil drop is $3.02 \times 10^{-5} \pm 0.05 \times 10^{-5}\ C$. THe data values are:

$$4.50 \times 10^{-3} \pm 0.10 \times 10^{-3} N$$
$$3.02 \times 10^{-5} \pm 0.05 \times 10^{-5}\ C$$

The quotient of the two data values:

$$E = \frac{F}{q}$$

$$\frac{4.50 \times 10^{-3} N}{3.02 \times 10^{-5} C} = 149. \frac{N}{C}$$

Error of the processed data values:

$$\frac{\Delta E}{E} = \frac{\Delta F}{F} + \frac{\Delta q}{q}$$

$$\frac{\Delta F}{F} = \frac{0.10 \times 10^{-3}\ N}{4.50\ \times 10^{-3}\ N} = 0.02222$$

$$\frac{\Delta q}{q} = \frac{0.05 \times 10^{-5}\ C}{3.02 \times 10^{-5}\ C} = 0.01656$$

$$\frac{\Delta E}{149.\frac{N}{C}} = 0.02222 + 0.01656$$

$$\Delta E = 6.\frac{N}{C}$$

Final data should be recorded as:

$$149. \pm 6.\frac{N}{C}$$

*Significant figures still apply to scientific notation, but don't affect the number of significant figures in a number.

Applying these rules to Conversions of a given quantity –

Ex:

A cork is determined to be 12.1 grams on a digital scale.

State this quantity and state the correct uncertainty in

A) Grams

B) Kg

C) weight in Newtons

Ans:

a) 12.1 grams +/0.1 grams

b) $12.1*10^{-3}$ kg +/- $0.1*10^{-3}$kg or

0.121 kg +/- .001 kg

c) 1.19N +/- 0.01 N

Practice Problems

Complete the following problems relating to the packet. Show work on a separate sheet of paper.

1. Determine the Error for the given device:
   a. Digital Gram Scale Accurate to 0.01 g _____
   b. Meter stick with 1mm increments _____
   c. Stop watch accurate to 0.01 s _____
   d. Stop Watch with a second-hand _____
2. Perform the given operation on the data values; include the error:
   a. Add: $2.64 \pm 0.01\ g$ and $13.08 \pm 0.01\ g$ _____

b. Add: $6.0 \pm 0.1\ m$ and $4.70 \pm 0.05\ g$ _____

c. Subtract: $20.16 \pm 0.10\ cm$ and $0.12 \pm 0.10\ cm$ _____

d. Subtract: $160. \pm 1.\ J$ and $45.4 \pm 0.5\ J$ _____

e. Multiply: $1.3 \times 10^3 \pm 0.1 \times 10^3\ kg$ and $12.5 \pm 0.6 \frac{m}{s}$ _____

f. Multiply: $154.3 \pm 0.1\ V$ and $4.76 \pm 0.10\ A$ _____

g. Divide: $309. \pm 8.\ N$ by $15.8 \pm .4 \frac{m}{s^2}$ _____

h. Divide: $521.6 \pm 1.5\ J$ by $24.96 \pm 0.01\ s$ _____

3. Applying these concepts to physics:
   a. A cube has a side length of $0.139 \pm 0.010\ m$. Determine the volume of this cube with the error.

$$V = s^3$$

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   b. An object with mass $120.0 \pm 1.0\ kg$ is experiencing a net acceleration of $2.80 \pm 0.05 \frac{m}{s^2}$. Calculate the force acting on the object with the error.

$$F = ma$$

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   c. Convert $1.73 \pm 0.10\ m$ into cm. Include the error.

$$100\ cm = 1\ m$$

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   d. *A car of mass $1200. \pm 50.\ kg$ is moving with a speed of $22.8 \pm 1.0 \frac{m}{s}$. Calculate the kinetic energy of the car with the error.

$$K.E. = \frac{1}{2}mv^2$$

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   e. **A ball of mass $0.155 \pm 0.001\ kg$ is being rotated in a circle, horizontally. The length of the string holding the ball is $.520 \pm 0.010\ m$, and the ball is moving at a speed of $2.68 \pm 0.05 \frac{m}{s}$. Calculate the force acting on the ball with the error.

$$F = \frac{mv^2}{r}$$

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4. Measuring and processing data:
   a. Measure and record the lengths of each side of the figure below. Record data in the corresponding chart. Calculate the length of each side from the measurements. All calculations should have the proper equation used and one example of data being calculated.

Side	Start Value	Error	End Value	Error	Length	Error
top						
right						
bottom						
left						

Figure:

b. Calculate the area and perimeter of this figure and their proper errors

Perimeter _____

Area _____

Answer Key – measurements and errors

1. Determined Error
   a. $\pm 0.01\ g$
   b. $\pm 0.5\ mm$
   c. $\pm 0.01\ s$
   d. $\pm 0.5\ s$
2. Solutions to the given operations and data
   a. $15.72 \pm 0.02\ g$
   b. $10.7 \pm 0.2\ g$
   c. $20.04 \pm 0.20\ cm$
   d. $115. \pm 2.\ J$
   e. $1.6 \times 10^4 \pm 0.2 \times 10^4 \frac{kg\ m}{s}$
   f. $734. \pm 16.\ W$
   g. $19.6 \pm 1.0\ kg$
   h. $20.90 \pm 0.07\ W$
3. Applied Concepts
   a. $2.69 \times 10^{-2} \pm 0.58 \times 10^{-2}\ m^3$
   b. $336. \pm 9.\ N$
   c. $173. \pm 10.\ cm$
   d. $3.12 \times 10^5 \pm 0.40 \times 10^5\ J$
   e. $2.14 \pm 0.13\ N$
4. Measuring and Processing Data
   a. Table

Side	Start Value	Error	End Value	Error	Length	Error
top	0.00 cm	0.05 cm	13.39 cm	0.05 cm	13.39cm	0.10 cm
right	0.00 cm	0.05 cm	6.80 cm	0.05 cm	6.80cm	0.10 cm
bottom	0.00 cm	0.05 cm	13.39 cm	0.05 cm	13.39cm	0.10 cm
left	0.00 cm	0.05 cm	6.80 cm	0.05 cm	6.80cm	0.10 cm

   b. Perimeter: $40.38 \pm 0.20\ cm$ Area: $91.1 \pm 2.0\ cm^2$